Latitude effect of the low momentum muon spectrum at sea level

1974 J. Phys. A: Math. Nucl. Gen. 7 150

(http://iopscience.iop.org/0301-0015/7/1/021)

J. Phys. A: Math., Nucl. Gen., Vol. 7, No. 1, 1974. Printed in Great Britain. © 1974

Latitude effect of the low momentum muon spectrum at sea level

A K De†, P Ghosh and A K Das

Department of Physics, University College of Science, Calcutta 9, India

Received 13 June 1973

Abstract. The sea level absolute vertical cosmic ray muon spectra at 12°N (Calcutta) have been measured in the range 0·35–2·2 GeV/*c* with a range spectrometer. These spectra are, on the average, 12 % higher than those of Bhattacharyya at the same place of observation. The lower value of intensities of Bhattacharyya might be due to underestimation of scattering corrections and rejections of a few per cent of 'single muons'. The measured differential spectrum may well be represented by a form-fit spectrum. The Durgapur spectrum of Nandi and Sinha at 12°N at sea level is in agreement with the form fit spectrum up to 20 GeV/*c*. A comparison of this spectrum with the high latitude form-fit spectrum of De *et al* gives 1·23, 1·25, 1·21 and 1·07 latitude effect at the momentum ranges 0·3–0·5, 0·5–1·0, 1·0–2·0 and 2·0–5·0 GeV/*c* respectively. These values are more or less in agreement with those predicted from Olbert. Above 5 GeV/*c* the latitude effect is observed to vanish.

1. Introduction

The vertical cosmic ray muon flux at sea level in the low momentum region is an important physical quantity which depends on the geomagnetic latitude. Olbert (1954) studied the latitude dependence of muon intensity on the basis of the high altitude measurements of Kraushaar (1949) and Conversi (1950). A number of authors (Basu and Sinha 1956–1957, Rose *et al* 1956, Fukui *et al* 1957, Subramanian *et al* 1958, Dau 1968, Allkofer *et al* 1972) have studied experimentally the latitude dependence of muon intensity. The results obtained by different authors except those of Subramanian *et al* are in agreement with those of Olbert's theoretical prediction. But the latitude effect (2·6–3·4) obtained by Subramanian *et al* between 50°N and 2°N at an altitude of 7500 ft above sea level seems to be too high in comparison with that predicted from Olbert's theory.

The above discrepancy together with the recent increase in muon intensities over those of Rossi (1948) at sea level by several authors (Allkofer *et al* 1970, Ayre *et al* 1971, Bateman *et al* 1971, Ashton *et al* 1972, Crookes and Rastin 1972) at high latitudes suggests that further studies of the geomagnetic latitude effect are of present interest.

An experiment for the absolute determination of muon intensities in the momentum range 0·35–2·2 GeV/*c* has been performed at 12°N at sea level with a range spectrometer to investigate whether the recent increase in muon intensities is also observed at low latitudes in contradiction to the absolute values at the same latitude (Bhattacharyya 1970, 1971, 1973, Dau 1968, Olbert 1954). Such an increase in muon intensity over the values obtained by the above authors at 12°N has already been reported by De *et al* (1972a, b) but only for a muon momentum near 1 GeV/*c*. The present data have been used for determination of the latitude effect when compared with the form-fit muon spectrum of De *et al* (1972b) for high latitudes.

---

† Also at B N College, Itachuna, Hooghly, India.

2. Experiment

The apparatus used in the present investigation was essentially a modification of that used in our previous communication (De *et al* 1972b). Figure 1 shows the front view of the apparatus. Two trays of crossed GM counters A and B, each with a sensitive area $7{\cdot}3 \times 7{\cdot}3\ \mathrm{cm}^2$ form a telescope with a geometric acceptance of $0{\cdot}4\ \mathrm{cm}^2\ \mathrm{sr}$. The small acceptance angle selects only the paraxial rays, and the probability of passing more than one muon at a time through the telescope would be very small. Below the counter tray B, there is an absorber L_1 of variable thicknesses 198–1468 $\mathrm{g\ cm}^{-2}$ lead equivalent including the roof and other materials above the apparatus to select different cut-off momenta. C and D are two plastic scintillation counters, each of area $50 \times 50\ \mathrm{cm}^2$, and in between them there is another absorber L_2 of thickness $194{\cdot}3\ \mathrm{g\ cm}^{-2}$ lead equivalent. A ring S of four trays of Geiger counters has been arranged around the telescope to account for the EAS events. The experiment was performed during the period July 1971 to January 1973 and the total time of observations was 387 days. Since each of the intensity measurements is made over a period of several weeks, the results are considered to correspond to the mean values of the temperature, pressure and 27 day variations of muon intensity. Moreover, since during the period of observation the phase of the solar cycle was in the region going to a minimum (nearer to the minimum), the measured intensities might have been increased at most 2 % in comparison with those of the eleven year mean values. This is because, it is known from the work of Forbush that the meson intensity changes by about 4 % during the course of the solar cycle. By simultaneous registration of count rates R_{ABC}, R_{ABCS}, R_{ABCD}, and R_{ABCDS} respectively of ABC, ABCS, ABCD and ABCDS

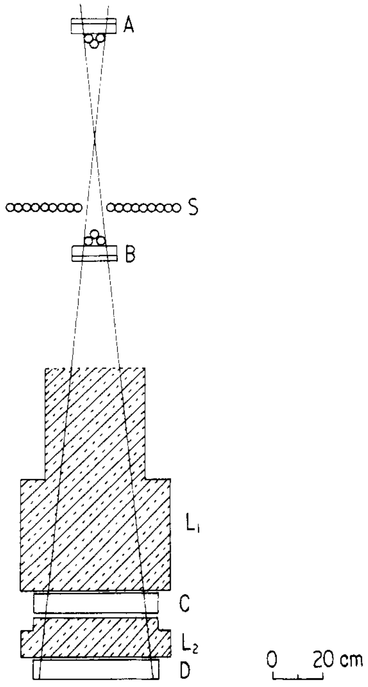

Figure 1. Front view of the apparatus, A, B, crossed Geiger counter arrays; S, Geiger counter shower tray; L_1, L_2, lead absorbers; C, D, scintillators.

coincidences, the two integral intensities at the absorber thicknesses L_1 (momentum p_1) and $L_1 + L_2 = L_{12}$ (momentum p_2) and one differential intensity at the mean absorber thickness can be determined. The exact differential muon rates $R_{ABC\overline{DS}}$ of ABC coincidence–DS anticoincidence are given by

$$R_{ABC\overline{DS}} = R_{ABC\bar{D}} - K_1(R_{ABC\bar{D}S} - K_2 R_{ABC\bar{D}}). \tag{1}$$

The factor K_1 was explained in De *et al* (1972a) and has the value 1·3.

The factor K_2 takes into account those single muons which produce knock-on electrons and hence are detected as showers. This was calculated from knock-on electron probabilities given by Bhabba (1938) and the values are given by 0·013, 0·031, 0·034, and 0·035 for muon momenta 0·5, 1·0, 1·5 and 2·0 GeV/*c* respectively.

All the rates have been corrected for the efficiency of each of the detectors, the dead time, the proton contribution and the scattering effect of the absorber.

The corrections for the loss of particles due to multiple Coulomb scattering inside the absorbers for integral counts have been accomplished by following the procedure described in De *et al* (1972b). The scattering corrections for differential counts were obtained from the difference of scattering corrections of two corresponding integral counts. The scattering corrections for the different absorber thicknesses were calculated and have been plotted in figure 2. The value of the scattering loss for the absorber L_{12} are much higher than that of the absorber L_1. This is because the particles are distributed in the absorber L_1 due to the scattering effect, and this effect increases on leaving this absorber and before entering the second absorber L_2.

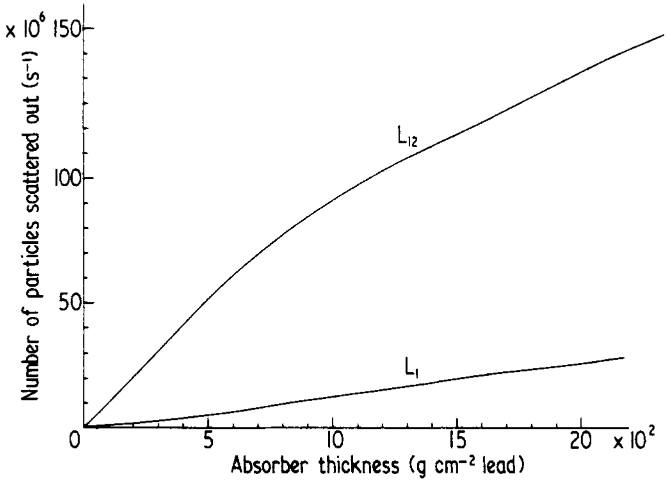

Figure 2. Variation of scattered-out particles with increase in the absorber thickness.

Due to the effect of the zig-zag motion of the particle inside the absorber, the absorber thicknesses, calculated according to Koenig (1946), had to be increased by 10% at 200 g cm^{-2} lead and 4% at 1700 g cm^{-2} lead. To find the momentum corresponding to different absorber thicknesses the table of Serre (1967) was used.

In this experiment the differential rate $R_{ABC\overline{DS}}$ does not include all muons with momenta between p_1 and p_2, and on the other hand, there are ABC$\overline{DS}$ muons with momenta outside this interval. This is because of the variation of the range of a muon due to fluctuation of energy loss (straggling) and in the zig-zag motion (multiple scattering). These two opposite effects practically compensate each other and give a correct momentum interval 99·7 % of (p_2-p_1) instead of (p_2-p_1).

3. Results and discussion

The results for the differential and integral muon intensities obtained in the present experiment have been plotted in figure 3. The present differential muon spectrum may well be represented (within 2 %) by the following form-fit spectrum:

$$N(p) = 2{\cdot}47\times10^{-3}p^{-0{\cdot}4854-0{\cdot}3406\ln(p)} \qquad \mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\,\mathrm{s}^{-1}(\mathrm{GeV}/c)^{-1}, \tag{2}$$

where p is in GeV/c and the constant $2{\cdot}47\times10^{-3}$ refers to the differential muon intensity at 1 GeV/c obtained from this experiment. The differential intensities of De *et al* (1972b) are in agreement with the form-fit spectrum. This spectrum is, on the average, 22 % higher than Olbert's (1954) spectrum at 12°N at sea level. It is also, on the average, 12 % higher than that of Bhattacharyya (1970) at the same place (12°N) and Dau (1968) at 9°N. However, as stated recently by Allkofer *et al* (1972) the intensities of Dau (1968) are not absolute. It is also interesting to note that the Durgapur spectrum of Nandi and Sinha (1972) is in agreement with the above form-fit spectrum in the range 5–20 GeV/c. The absolute value of muon intensity of Basu and Sinha (1956–1957) at 12°N at sea level

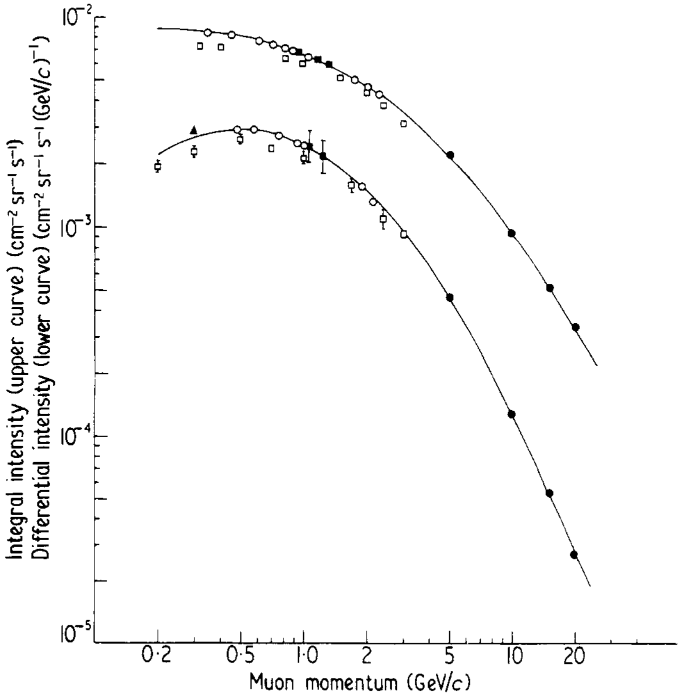

Figure 3. The best-fit muon spectra together with experimental points at 12°N at sea level. ○ Present experiment; — form-fit spectrum at 12°N; ■ De *et al* (1972a, b); ○ Nandi and Sinha (1972); N Bhattacharyya (1970, 1971, 1973); ▲ Basu and Sinha (1956–1957).

at 0·30 GeV/c is $(2{\cdot}89 \pm 0{\cdot}1) \times 10^{-3}$ cm^{-2} sr^{-1} s^{-1} (GeV/c)$^{-1}$ and is in agreement with the above form-fit.

Other evidence in support of the form-fit spectrum given in equation (2), is that the integral intensities obtained from this experiment are in agreement within 2% (figure 3) with the integral form spectrum given by

$$I(p) = I(10) + \int_{p}^{10} N(p)\,\mathrm{d}p \tag{3}$$

where the normalization point $I(10) = 0{\cdot}94 \times 10^{-3}$ cm^{-2} sr^{-1} s^{-1} is the integral intensity of muons at 10 GeV/c obtained by Nandi and Sinha (1972) at 12°N at sea level. The integral intensities of De *et al* (1972a, b) also lie on the form-fit spectrum. Thus the form-fit spectra given by equations (2) and (3) are practical formulae for the computation of differential and integral muon rates at 12°N for momenta below 10 GeV/c. The integral spectrum is, on the average, 11% higher than that of Bhattacharyya (1973) at the same latitude. Bhattacharyya (1973) has fitted his low momentum integral muon intensities with the phenomenological form given by Barrett *et al* (1952) following the procedure of Allkofer *et al* (1971). Keeping other parameters the same as those used by Allkofer *et al* (1971) he has obtained $A = 0{\cdot}169$ to fit his experimental results. This value of A seems to be low in comparison with that determined by himself ($A = 0{\cdot}206$) (Bhattacharyya 1972) for the same experimental data and also those obtained by the other authors.

In his measurements, Bhattacharyya (1970, 1971, 1973) has used range spectrographs using flash tubes for track location. The lower values of intensities of Bhattacharyya (1970, 1971, 1973) in comparison to that of the present experiments may be attributed to the following defects of his apparatus and in the method of analysis: (i) the lead absorbers were placed immediately above the flash-tube stacks. This arrangement would produce flash-tube pictures in which some single muon tracks were accompanied by low energy electrons produced by muons at the end of the absorber. As stated by Bhattacharyya (1973) these events were taken as 'accompanied particles' instead of 'single particles'. (ii) As the telescope was narrow the lateral displacement due to multiple Coulomb scattering inside the absorber should increase the scattering loss. The absorber was also not sufficiently extended and as such there was very small scattering-in effect in comparison with the scattering-out effect. Therefore, the net scattering loss effect should not be small for the apparatus of Bhattacharyya. Two sets of absorbers Σ and T should further increase the scattering loss of particles. This is because the particles are distributed inside Σ and this effect increases on leaving the first absorber and before entering the second one T. Further, the telescope used by Bhattacharyya was similar to that used by Greisen (1942). The scattering loss for Greisen's experiment as calculated by Crooks and Rastin (1972) is found to be 18·3%. If the entire space in between the two extreme counters of Greisen's apparatus was filled with lead absorber (~25 cm) the scattering loss should be further increased. Therefore, considering the dimensions of the telescope used by Bhattacharyya one may conclude that the scattering corrections applied to his experimental results were very small (0·4–1·7% for different absorber thicknesses) in comparison to the actual values. (iii) Lastly, he made a very simple and serious mistake in his experiments (Bhattacharyya 1970, 1971, 1973) by not considering the effect of the roofs which were above his telescope. For this there should be an increase in absorber thickness of at least 100 g cm^{-2} of lead equivalent and for which the integral intensities would have been increased by 3% and 5·3% at 0·32 and 1 GeV/c

respectively. From the above discussions it may be stated that the lower values of intensities of Bhattacharyya (1970, 1971, 1973) were due to underestimation of scattering loss corrections and rejection of a few per cent of single muons.

Most of the results of recent absolute measurements of muon intensities at high latitudes in the low momentum region agree (within 8 %) with the following form-fit spectrum presented by De *et al* (1972b):

$$N(p) = 3{\cdot}09 \times 10^{-3} p^{-0{\cdot}5483 - 0{\cdot}3977 \ln(p)} \quad \mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\,\mathrm{s}^{-1}\,(\mathrm{GeV}/c)^{-1}. \tag{4}$$

The spectrum is normalized at the standard momentum of 1 GeV/*c* to the value

$$3{\cdot}09 \times 10^{-3}\,\mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\,\mathrm{s}^{-1}(\mathrm{GeV}/c)^{-1}$$

obtained by Allkofer *et al* (1970) with the help of a similar experimental arrangement to that of the present one. Allkofer and Jokisch (1973) stated that equation (4) is the most satisfactory fit to the high latitude results of muon intensities in the low momentum region.

A comparison of the two form-fit spectra (equations (2) and (4)) gives on the average, 1·23, 1·25, 1·21 and 1·07 latitude effects between 53°N and 12°N (latitude effect is defined as the ratio of the two intensities) in the momentum ranges 0·3–0·5, 0·5–1·0, 1·0–2·0 and 2·0–5·0 GeV/*c* respectively and have been plotted in figure 4. The latitude effect is observed to vanish above 5 GeV/*c*. It should be mentioned that equation (4) corresponds to a muon spectrum at solar maximum whereas equation (2) corresponds to the same near the solar minimum. Hence the measured latitude effects might have been decreased by at most 5 % compared with those of the eleven year mean values. Also in figure 4 the latitude effect of the muon spectrum between 45°N and 12°N obtained from the theoretical calculation of Olbert (1954) has been plotted. It is observed that the present values of the latitude effect of muon intensities are more or less in agreement with those predicted from Olbert (1954). The latitude effects of muon intensities between 53°N (Kiel) and the equator at sea level were studied by Dau (1968) and Allkofer *et al* (1972) using the same magnetic spectrograph so that the bias effect of the instrument was eliminated. The results have been plotted in figure 4 and are found to be in agreement

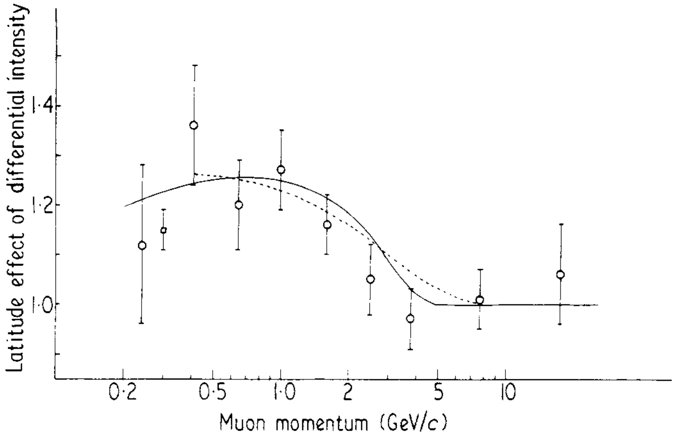

Figure 4. Latitude effect of differential muon spectrum at sea level. — Present work; --- Olbert (1954); □ Basu and Sinha (1956–1957); ○ Dau (1968).

with the present values. It should be noted that the absolute intensities of Dau (1968) are lower than the recent absolute values of muon intensities at high latitudes. The spectra of Dau (1968) at low latitude, though in agreement with Bhattacharyya (1970, 1971, 1973), have recently been rejected by Allkofer *et al* (1972) in respect of absolute values. Basu and Sinha (1956–1957) also obtained the latitude effect of sea level muons at 0·3 GeV/*c* by comparing their differential muon intensity at 12°N with that of Brini *et al* (1955) at 45°N. Their value $1\cdot15 \pm 0\cdot04$ is in reasonable agreement with that of the present work.

It is to be noticed from figure 4 that the latitude effect obtained from the present work and from that of Dau (1968) increases with decreasing momentum up to 0·5 GeV/*c*, but below this momentum the effect decreases with decreasing momentum. This is due to the fact that comparatively high energy primaries ($\geqslant$ 10 GeV/*c*) produce charged secondaries which can reach sea level before being absorbed (Wolfendale 1963). The primary particles in the momentum range 10–15 GeV/*c* (15 GV is the cut-off rigidity at low latitude) are responsible for higher values of intensities at sea level at high latitude in comparison with those at low altitude; ie for the latitude effect of sea level muons. Sea level muons produced by the primary particles of momenta 10–15 GeV/*c* are distributed with a flat maximum in the region 0·5–1·2 GeV/*c*. This distribution of sea level muons decreases both below and above 0·5 and 1·2 GeV/*c* respectively. Thus the latitude effect of the differential muon intensity at sea level is a maximum in the region 0·5–1·2 GeV/*c* and decreases below 0·5 GeV/*c* and above 1·2 GeV/*c*. In this connection the result for the latitude effect obtained by Subramanian *et al* (1958) at 2°N should be mentioned. Comparing the value of muon intensity obtained by Subramanian *et al* at 0·22 GeV/*c* at 7500 ft above sea level with that at 50°N at the same momentum and altitude they obtained a latitude effect of 3·0. This value seems to be too high when compared with the value of about 1·35 expected from a knowledge of the variation of latitude effect with altitude.

The latitude effects of integral muon intensities have also been determined by comparing equation (3) with the corresponding equation for the integral intensity as given by Allkofer and Jokisch (1973) and have been plotted in figure 5. The experimental points of Dau (1968), Rose *et al* (1956) and the theoretical curves of Olbert (1954) have also been plotted in figure 5. As seen in figure 5 the values of the latitude effect obtained by different authors are in good agreement with the present results.

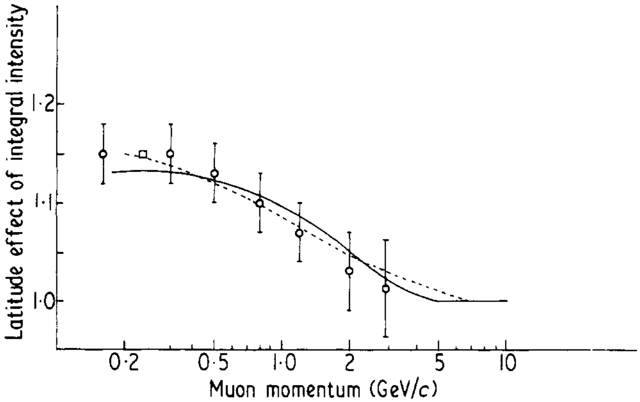

Figure 5. Latitude effect of integral muon spectrum at sea level. — Present work; - - - Olbert (1954); □ Rose *et al* (1956); ○ Dau (1968).

4. Conclusion

Olbert's theoretical predictions concerning the latitude effect of muon intensities are in agreement with the results of the present report.

The present work provides reliable cosmic ray muon spectra at 12°N at sea level in the low momentum region.

Acknowledgments

The authors wish to record their gratitude to Professor P C Bhattacharya for his interest in the work. Thanks are due to S Mitra for helpful discussions. Technical assistance received from the technical staff (including workshops) of the Physics Department is gratefully acknowledged. One of us (PG) is indebted to the University Grants Commission, India for the award of a research scholarship.

The computer centre of Calcutta University is also thanked for providing us computational facilities.

References

Allkofer O C, Andresen R D, Clausen K and Dau W D 1972 *J. Geophys. Res.* 77 4251–3
Allkofer O C, Dau W D and Jokisch H 1970 *Proc. 6th Interam Sem. on Cosmic Rays, La Paz* (La Paz: University of San Andreas) pp 937–48
Allkofer O C *et al* 1971 *Phys. Lett.* **36B** 425–7
Allkofer O C and Jokisch H 1973 *Nuovo Cim.* A **15** 371–89
Ashton F, Tsuji K and Wolfendale A W 1972 *Nuovo Cim.* B **9** 344–50
Ayre C A *et al* 1971 *J. Phys.* A: *Gen. Phys.* **4** L89–93
Barrett P H *et al* 1952 *Rev. mod. Phys.* **24** 133–78
Basu N and Sinha M S 1956–1957 *Trans. Bose Res. Inst.* **21** 67–79
Bateman B J *et al* 1971 *Phys. Lett.* **36B** 144–8
Bhabha H J 1938 *Proc. R. Soc.* A **164** 257–94
Bhattacharyya D P 1970 *Z. Phys.* **234** 17–22
—— 1971 *PhD Thesis* Calcutta University
—— 1972 *Z. Phys.* **257** 292–8
—— 1973 *J. Phys.* A: *Math., Nucl. Gen.* **6** 582–7
Brini D *et al* 1955 *Nuovo Cim.* **2** 613
Conversi M 1950 *Phys. Rev.* **79** 750–67
Crookes J N and Rastin B C 1972 *Nucl. Phys.* B **39** 493–508
Dau W D 1968 *PhD Thesis* University of Kiel
De A K *et al* 1972a *Phys. Rev.* D **5** 1068–72
—— 1972b *J. Phys.* A: *Gen. Phys.* **5** 1236–42
Fukui S, Kitmura T and Murata Y 1957 *J. Phys. Soc. Japan* **12** 854–63
Greisen K 1942 *Phys. Rev.* **61** 212–21
Koenig H P 1946 *Phys. Rev.* **69** 590–6
Kraushaar W L 1949 *Phys. Rev.* **76** 1045–58
Nandi B C and Sinha M S 1972 *J. Phys.* A: *Gen. Phys.* **5** 1384–94
Olbert S 1954 *Phys. Rev.* **96** 1400–7
Rose D C *et al* 1956 *Can. J. Phys.* **34** 968–84
Rossi B 1948 *Rev. mod. Phys.* **20** 537–83
Serre C 1967 *CERN Report* 67–5 pp 1–40
Subramanian A *et al* 1958 *Nuovo Cim.* **7** 110–3
Wolfendale A W 1963 *Cosmic Rays* (London: George Newnes) p 130